

Wormhole as the end state of two-dimensional black hole evaporation

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ABSTRACT

We present a specific two-dimensional dilaton gravity model in which a black hole evaporates leaving a wormhole at the end state. As the black hole formed by infalling matter in a initially static spacetime evaporates by emitting Hawking radiation, the black hole singularity that is initially hidden behind a timelike apparent horizon meets the shrinking horizon. At this intersection point, we imposed boundary conditions which require disappearance of the black hole singularity and generation of the exotic matter which is the source of the wormhole as the end state of the black hole. These, of course, preserve energy conservation and continuity of the metric.

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Hawking's discovery[1] that black holes radiate thermally was the beginning of applications of quantum mechanics in black hole physics. But this discovery has raised the information problem: according to Hawking's calculation, when a black hole formed in collapse of a pure state evaporates, the resulting outgoing state is approximately thermal and in particular is a mixed state. This conflicts with the ordinary laws of quantum mechanics which always preserve purity. This disagreement can be avoided by introducing the final geometry resulting from black hole evaporation to preserve purity.

In this paper we present a specific two-dimensional dilaton gravity model[2] in which a black hole evaporates leaving a wormhole as the end state. Recently Hayward[3] proposed that black holes and wormholes are interconvertible. In particular, if a wormhole's negative-energy generator fails or the negative-energy source is overwhelmed by normal ordinary positive-energy matter, it will become a black hole, and a wormhole could be constructed from a suitable black hole by irradiating it with negative energy.

The classical two-dimensional Callan-Giddings-Harvey-Strominger(CGHS) action[4] is

$$\begin{aligned} S_{\text{cl}} &= S_{\text{G}} + S_{\text{M}} \\ &= \frac{1}{2\pi} \int d^2x \sqrt{-g} \left(e^{-2\phi} \left[R^{(2)} + 4(\nabla\phi)^2 + 4\lambda^2 \right] - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right), \end{aligned} \quad (1)$$

where ϕ is a dilaton field, $R^{(2)}$ is the 2D Ricci scalar, λ is a positive constant, ∇ is the covariant derivative, and the f_i are N matter (massless scalar) fields. This action admits vacuum solutions, static black hole solutions, dynamical solutions describing the formation of a black hole by collapsing matter fields and wormhole solutions as we will see later.

To see one-loop quantum effects and back reaction, one can use the trace anomaly, $\langle T^\mu{}_\mu \rangle = (\hbar/24)R^{(2)}$, for massless scalar fields in two dimensions and the Polyakov-Liouville action,

$$S_{\text{PL}} = -\frac{\hbar}{96\pi} \int d^2x \sqrt{-g(x)} \int d^2x' \sqrt{-g'(x')} R^{(2)}(x) G(x, x') R^{(2)}(x'), \quad (2)$$

for which

$$\langle T^{\mu\nu} \rangle = -\frac{2\pi}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} S_{\text{PL}}, \quad (3)$$

where $G(x, x')$ is a Green function for ∇^2 . Here we take the large- N limit, in which \hbar goes to zero while $N\hbar$ is held fixed. In that limit the quantum corrections for the gravitational and dilaton fields are negligible, and one needs to take into account only the quantum corrections for the matter fields. The one-loop effective action is then $S_{(1)} = S_{\text{cl}} + NS_{\text{PL}}$. In order to find analytic solutions including semiclassical corrections, one can modify the action as in Refs. [5]-[7]. We use the model modified from the original CGHS model by Bose, Parker and Peleg[2].

They added to the classical action (1) a local covariant term of one-loop order:

$$S_{\text{corr}} = \frac{N\hbar}{24\pi} \int d^2x \sqrt{-g} [(\nabla\phi)^2 - \phi R^{(2)}]. \quad (4)$$

Now the total modified action including the one-loop Polyakov-Liouville term is

$$S = S_{\text{cl}} + S_{\text{corr}} + NS_{\text{PL}}. \quad (5)$$

We use the (extended) null coordinates $x^\pm = x^0 \pm x^1$ (the both coordinates x^+ and x^- cover the entire range $(-\infty, +\infty)$) and the conformal gauge $g_{++} = g_{--} = 0$, $g_{+-} = -\frac{1}{2}e^{2\beta}$. We can choose $\phi(x^+, x^-) = \beta(x^+, x^-)$ in analyzing the equations of motion. In the conformal gauge the equations of motion derived from S are the same as the classical ones

$$\begin{aligned} \partial_{x^+} \partial_{x^-} (e^{-2\beta(x^+, x^-)}) &= \partial_{x^+} \partial_{x^-} (e^{-2\phi(x^+, x^-)}) = -\lambda^2, \\ \partial_{x^+} \partial_{x^-} f_i(x^+, x^-) &= 0, \end{aligned} \quad (6)$$

while the constraints are modified by nonlocal terms $t_\pm(x^\pm)$ arising from the Polyakov-Liouville action. In conformal gauge, one can use the trace anomaly of N massless scalar fields f_i to obtain $\langle T_{+-}^f \rangle = -\kappa \partial_{x^+} \partial_{x^-} \beta$, where $\kappa = N\hbar/12$, and integrate the equation $\nabla^\mu \langle T_{\mu\nu}^f \rangle = 0$ to get the quantum corrections to the energy-momentum tensor of the f_i matter fields:

$$\langle T_{\pm\pm}^f \rangle = \kappa [\partial_{x^\pm}^2 \beta - (\partial_{x^\pm} \beta)^2 - t_\pm(x^\pm)], \quad (7)$$

where $t_\pm(x^\pm)$ are integration functions determined by boundary conditions. And the modified constraints $\frac{\delta S}{\delta g^{\pm\pm}} = 0$ become

$$-\partial_{x^\pm}^2 (e^{-2\phi(x^+, x^-)}) - (T_{\pm\pm}^f)_{\text{cl}} + \kappa t_\pm(x^\pm) = 0, \quad (8)$$

where $(T_{\pm\pm}^f)_{\text{cl}} = \frac{1}{2} \sum_{i=1}^N (\partial_{x^\pm} f_i)^2$ is the classical contribution to the energy-momentum tensor of the f_i matter fields.

For a given classical matter distribution and a given $t_\pm(x^\pm)$ one finds the solution for the equations of motion (6) with constraints (8),

$$\begin{aligned} e^{-2\phi} = e^{-2\beta} = & -\lambda^2 x^+ x^- - \int^{x^+} dx_2^+ \int^{x_2^+} dx_1^+ [(T_{++}^f)_{\text{cl}} - \kappa t_+(x_1^+)] \\ & - \int^{x^-} dx_2^- \int^{x_2^-} dx_1^- [(T_{--}^f)_{\text{cl}} - \kappa t_-(x_1^-)] + a_+ x^+ + a_- x^- + b, \end{aligned} \quad (9)$$

where a_\pm and b are constants. In the choice $(T_{\mu\nu}^f)_{\text{cl}} = 0$ and $t_\pm(x^\pm) = a_\pm = b = 0$, it means the linear dilaton flat spacetime solution.

It also has static black hole solutions corresponding to choice $(T_{\mu\nu}^f)_{\text{cl}} = 0$, $t_\pm(x^\pm) = a_\pm = 0$ and $b = M/\lambda$. In order to find the solution corresponding to the Minkowski vacuum asymptotically, one can use Eq.(7) to find the solution for which $\langle T_{\pm\pm}^f(\sigma^\pm) \rangle = 0$ in flat coordinates σ^\pm . The functions $t_\pm(x^\pm)$ are determined by imposing appropriate boundary conditions that the metric is flat, such that β and its derivatives vanish in the asymptotically flat coordinates σ^\pm . Then we get

$$\langle T_{\pm\pm}^f(\sigma^\pm) \rangle|_{\text{boundary}} = -\kappa t_\pm(\sigma^\pm) = 0, \quad (10)$$

and

$$t_\pm(x^\pm) = \left(\frac{\partial \sigma^\pm}{\partial x^\pm} \right) t_\pm(\sigma^\pm) - \frac{1}{2} D_{x^\pm}^S[\sigma^\pm] = \frac{1}{(2x^\pm)^2}, \quad (11)$$

where $D_y^S[z]$ is the Schwarzian derivative $D_y^S[z] = \partial_y^3 z / (\partial_y z) - \frac{3}{2} (\partial_y^2 z / \partial_y z)^2$ and we use the fact that the Minkowski vacuum corresponds to Eq.(10). Thus we find that the asymptotically Minkowski-vacuum solution is

$$e^{-2\phi} = e^{-2\beta} = -\lambda^2 x^+ x^- - \frac{\kappa}{4} \ln |\lambda^2 x^+ x^-| + C, \quad (12)$$

where C is a constant. This solution has two apparent horizons in the extended coordinates.

Next we turn to the dynamical scenario in which the spacetime is initially described by one of the static solutions in Eq.(12), and in which the black holes are formed by collapsing matter fields, particularly the simple shock wave of infalling matter described

by $(T_{++}^f)_{\text{cl}} = (M/\lambda x_0^+) \delta(x^+ \pm x_0^+)$, where $x_0^+ > 0$, and $(T_{--}^f)_{\text{cl}} = 0$. Then we find the solution

$$e^{-2\phi} = e^{-2\beta} = -\lambda^2 x^+ x^- - \frac{\kappa}{4} \ln |\lambda^2 x^+ x^-| - \frac{M}{\lambda x_0^+} (x^+ - x_0^+) \Theta(x^+ \pm x_0^+) + C, \quad (13)$$

where

$$\begin{aligned} \Theta(x^+ \pm x_0^+) &= 0 & -x_0^+ \leq x^+ \leq x_0^+ \\ &= 1 & \text{elsewhere.} \end{aligned} \quad (14)$$

For all values of M and C , the solution after shock wave is

$$e^{-2\phi} = e^{-2\beta} = -\lambda^2 x^+ (x^- + \Delta) - \frac{\kappa}{4} \ln |\lambda^2 x^+ x^-| + \frac{M}{\lambda} + C, \quad (15)$$

where $\Delta = M/(\lambda^3 x_0^+)$. The black hole singularity curve is

$$-\lambda^2 x_s^+ (x_s^- + \Delta) - \frac{\kappa}{4} \ln |\lambda^2 x_s^+ x_s^-| + \frac{M}{\lambda} + C = 0, \quad (16)$$

and an apparent horizon, defined by $\partial_{x^+} e^{-2\phi} = 0$ [5], is

$$-\lambda^2 x_h^+ (x_h^- + \Delta) = \frac{\kappa}{4}. \quad (17)$$

When the apparent horizon is formed, the black hole starts radiating. At future null infinity \mathcal{I}^+ one can calculate Hawking radiation in the asymptotically flat coordinates $\hat{\sigma}^\pm$, defined by $\lambda \hat{\sigma}^+ = \ln |\lambda x^+|$ and $-\lambda \hat{\sigma}^- = \ln |\lambda (x^- + \Delta)|$,

$$\langle T_{--}^f(\hat{\sigma}^\pm) \rangle|_{\mathcal{I}^+} = \frac{\kappa \lambda^2}{4} \left[1 - \frac{1}{(1 + \lambda \Delta e^{\lambda \hat{\sigma}^-})^2} \right]. \quad (18)$$

Initially the singularity is behind apparent horizon, but as the black hole evaporates by emitting Hawking radiation the apparent horizon shrinks and eventually meets the singularity at $(x_{\text{int}}^+, x_{\text{int}}^-)$,

$$\begin{aligned} x_{\text{int}}^+ &= \frac{1}{\lambda^2 \Delta} \left[\pm \exp \left(\frac{4(\pm M + \lambda C)}{\kappa \lambda} + 1 \right) - \frac{\kappa}{4} \right], \\ x_{\text{int}}^- &= -\Delta \left[1 \mp \frac{\kappa}{4} \exp \left(-\frac{4(\pm M + \lambda C)}{\kappa \lambda} - 1 \right) \right]^{-1}, \end{aligned} \quad (19)$$

where the upper signs do when $x^+ x^- < 0$ and the lower when $x^+ x^- > 0$. The singularity become naked after the singularity and the apparent horizon have merged. The

future evolution is not uniquely determined unless boundary conditions are imposed at the naked singularity[5].

Here we consider the solution to the future of the point $(x_{\text{int}}^+, x_{\text{int}}^-)$; at the intersection point the evaporating black hole matches to a stable wormhole keeping continuous metric and energy conservation. We find the boundary conditions that match the solution (15) continuously to a static wormhole solution.

Before this matching, we present wormhole solutions in the (1+1)-dimensional dilaton gravity. The equations of motion from the classical action (1) are

$$\frac{2}{\pi}e^{-2\phi} \left[\nabla_\mu \nabla_\nu \phi + g_{\mu\nu} \left((\nabla\phi)^2 - \nabla^2\phi - \lambda^2 \right) \right] - T_{\mu\nu} = 0 \quad (20)$$

$$e^{-2\phi} \left[R + 4\lambda^2 + 4\nabla^2\phi - 4(\nabla\phi)^2 \right] = 0, \quad (21)$$

where the first equation is derived from variation of the metric and the second is the dilaton equation of motion.

To find the traversable wormhole solution of this theory we first introduce the spacetime metric[8]

$$ds^2 = -e^{2\Phi(r)} dt^2 + \left(1 - \frac{b(r)}{r} \right)^{-1} dr^2, \quad (22)$$

where $\Phi(r)$ is redshift function which determines the structure of the wormhole and defined by

$$g_{tt} = -e^{2\Phi}, \quad (23)$$

and $b(r)$ is the shape function that relates to the proper radial distance $l(r)$ from wormhole throat by

$$dl/dr = \pm (1 - b/r)^{-1/2}. \quad (24)$$

In order for the spatial geometry to tend to an appropriate asymptotically flat limit, $\lim_{r \rightarrow \infty} b(r)$ must be finite. By comparison with the Schwarzschild metric this implies that the mass of the wormhole, as seen from spatial infinity, is given by $\lim_{r \rightarrow \infty} b(r) = 2GM_w$, where M_w is ADM mass of the wormhole[9].

To produce a traversable wormhole several general constraints on $b(r)$ and $\Phi(r)$ are required. The first constraint is that the spatial geometry must have wormhole shape, i.e., two flat regions and a narrow one. Thus throat is at minimum of $r = b = b_0$, $1 - b/r \geq 0$ throughout spacetime, and $b/r \rightarrow 0$ as $l \rightarrow \pm\infty$ (asymptotically flat

regions of two universes) so $r \cong |l|$. This constraint means that the embedding surface flares outward, forcing $(d^2r/dz^2) > 0$ so that $r(z)$ is a minimum at the throat in two-dimensional embedding Euclidean space with coordinates z and r . Consequently, we have

$$\frac{d^2r}{dz^2} = \frac{b - b'r}{2b^2} > 0. \quad (25)$$

Secondly, there should be no horizons or singularities since a horizon would prevent two-way travel through the wormhole, so Φ is everywhere finite and $\Phi \rightarrow 0$ as $l \rightarrow \pm\infty$. The last constraint is following: the matter and field that generate the spacetime curvature for the wormhole must have a physically reasonable stress-energy tensor with non-zero components $T_{\hat{t}\hat{t}}$ and $T_{\hat{r}\hat{r}}$. In the proper reference frame of a set of observers who remain always at rest in the coordinate system

$$\begin{aligned} \mathbf{e}_{\hat{t}} &= e^{-\Phi} \mathbf{e}_t \\ \mathbf{e}_{\hat{r}} &= (1 - b/r)^{1/2} \mathbf{e}_r, \end{aligned} \quad (26)$$

the non-zero components of the stress-energy tensor are $T_{\hat{t}\hat{t}} = \rho =$ density of mass-energy and $T_{\hat{r}\hat{r}} = -\tau = -$ radial tension. In this basis the metric coefficients take on the Minkowskian forms,

$$g_{\hat{\alpha}\hat{\beta}} = \mathbf{e}_{\hat{\alpha}} \cdot \mathbf{e}_{\hat{\beta}} = \eta_{\hat{\alpha}\hat{\beta}} \equiv \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (27)$$

The spacetime metric (22) can be written like followings by introducing (extended) light-cone coordinates $y^\pm = y^0 \pm y^1$ (y^\pm also cover $(-\infty, +\infty)$), and by choosing conformal gauge $g_{\mu\nu} = e^{2\beta} \eta_{\mu\nu}$,

$$\begin{aligned} ds^2 &= -e^{2\Phi} [dt^2 - dr^{*2}] \\ &= -e^{2\beta} dy^+ dy^- \end{aligned} \quad (28)$$

where $r^* = \int e^{-\Phi} \left(1 - \frac{b}{r}\right)^{-\frac{1}{2}} dr$. As r grows very large, $r^* \rightarrow \infty$. On the other hand, when r approaches $b = b_0$ then $r^* \rightarrow 0$ since Φ is everywhere finite and we can set $dr^* \rightarrow \infty$ at $r^* = 0$. We then have the nonzero components of energy-momentum tensor in (extended) light-cone coordinates,

$$T_{+-} = \frac{\partial t}{\partial y^+} \frac{\partial t}{\partial y^-} T_{tt} + \frac{\partial r}{\partial y^+} \frac{\partial r}{\partial y^-} T_{rr}$$

$$\begin{aligned}
&= e^{2\beta}(\rho + \tau) \\
T_{++} &= \frac{\partial t}{\partial y^+} \frac{\partial t}{\partial y^+} T_{tt} + \frac{\partial r}{\partial y^+} \frac{\partial r}{\partial y^+} T_{rr} \\
&= -e^{2\beta} \frac{y^-}{y^+} (\rho - \tau) \\
T_{--} &= \frac{\partial t}{\partial y^-} \frac{\partial t}{\partial y^-} T_{tt} + \frac{\partial r}{\partial y^-} \frac{\partial r}{\partial y^-} T_{rr} \\
&= -e^{2\beta} \frac{y^+}{y^-} (\rho - \tau).
\end{aligned} \tag{29}$$

And the conservation law of energy-momentum tensor, $\nabla^\mu T_{\mu\nu} = 0$, gives

$$\rho e^{2\beta} = \frac{c}{|\lambda^2 y^+ y^-|}, \tag{30}$$

where c is a constant. In conformally invariant case ($T_{+-} = 0$), the equations of motion become

$$\frac{2}{\pi} e^{-2(\phi+\beta)} \left[-\partial_{y^+} \partial_{y^-} \phi + 2\partial_{y^+} \phi \partial_{y^-} \phi + \frac{1}{2} \lambda^2 e^{2\beta} \right] = \rho + \tau = 0 \tag{31}$$

$$e^{-2\phi} \left[\partial_{y^+} \partial_{y^-} \beta - 2\partial_{y^+} \beta \partial_{y^-} \phi + 2\partial_{y^+} \phi \partial_{y^-} \phi + \frac{1}{2} \lambda^2 e^{2\beta} \right] = 0. \tag{32}$$

Since we have gauge fixed g_{++} and g_{--} to zero, we should impose their equations of motion as constraints in conformally invariant case,

$$\frac{2}{\pi} e^{-2\phi} (2\partial_{y^+} \beta \partial_{y^+} \phi - \partial_{y^+}^2 \phi) = 2\rho e^{2\beta} \frac{y^-}{y^+} \tag{33}$$

$$\frac{2}{\pi} e^{-2\phi} (2\partial_{y^-} \beta \partial_{y^-} \phi - \partial_{y^-}^2 \phi) = 2\rho e^{2\beta} \frac{y^+}{y^-}. \tag{34}$$

So we get the solution determined in choice of $\phi = \beta$ as

$$e^{-2\phi} = e^{-2\beta} = C' + C_0 \ln |\lambda^2 y^+ y^-| + C_1 y^+ + C_2 y^- - \lambda^2 y^+ y^-, \tag{35}$$

where C', C_0, C_1 and C_2 are constants and we can choose c in Eq.(30) so that $C_0 = \frac{2\pi c}{\lambda^2} = -\frac{\kappa}{4}$. For a static wormhole the constants C_1 and C_2 should be zero,

$$e^{-2\phi} = e^{-2\beta} = C' - \frac{\kappa}{4} \ln |\lambda^2 y^+ y^-| - \lambda^2 y^+ y^-. \tag{36}$$

Because the wormhole has forbidden range ($r < b_0$) we have to restrict the ranges of y^+ and y^- such as $|\lambda^2 y^+ y^-| \geq 1$. In the regions of $-\lambda^2 y^+ y^- \geq 1$, we have to fix

this solution with $C' > -1$ and $\frac{\kappa}{4} < 1$ since wormhole solution has no singularities and horizons. In the other regions, $C' > 1$ is required from the same reason. In this wormhole solution, one can let $C' = \frac{M_w}{\lambda}$ like the black hole case.

Since the physical meaning of the wormhole solution is better in coordinates where the metric is asymptotically constant on \mathcal{I}_R^+ , we set that

$$\begin{aligned}\lambda y^+ &= \pm e^{\lambda \sigma^+} \\ \lambda y^- &= \pm e^{-\lambda \sigma^-},\end{aligned}\tag{37}$$

where $\sigma^\pm = t \pm r^*$. These constraints preserve the conformal gauge and gives

$$e^{-2\beta} = C' - \frac{\kappa}{2}\lambda r^* + e^{2\lambda r^*}.\tag{38}$$

Thus, with the proper asymptotic flatness,

$$e^{-2\Phi(r^*)} = C' e^{-2\lambda r^*} - \frac{\kappa}{2}\lambda r^* e^{-2\lambda r^*} + 1\tag{39}$$

$$\rho(r^*) = -\tau(r^*) = c [C' e^{-2\lambda r^*} - \frac{\kappa}{2}\lambda r^* e^{-2\lambda r^*} + 1]\tag{40}$$

As r grows very large, i.e., $r^* \rightarrow \infty$, $\Phi = 0$ and $\rho = c$. And when r approaches $b = b_0$ then $r^* \rightarrow 0$ as we mentioned before, so at the throat

$$\rho = c (C' + 1) \equiv \rho_0\tag{41}$$

$$\Phi = -\frac{1}{2} \ln(C' + 1),\tag{42}$$

where $c < 0$ so that the constant ρ_0 is a large negative value, since the exotic matter that violates the null energy condition would be concentrated near the throat for almost traversable wormhole. From a classical perspective, violation of the null energy condition is not permitted. However we know that some quantum effects lead to measurable experimentally verified violations of the null energy condition. In particular, Hawking evaporation violates the area increase theorem for classical black holes. This implies that the quantum processes underlying the Hawking evaporation process must also induce a violation of the input assumption (the null energy condition - the only input assumption that seems weak in quantum violation) used in proving the classical area increase theorem[9].

Now we give the boundary conditions that match the evaporating black hole solution (15) to the static wormhole solution (36) continuously at the intersection points $(x_{\text{int}}^+, x_{\text{int}}^-)$. The amount of energy E_{rad} radiated by the black hole up to the curve $x^- = x_{\text{int}}^-$ is calculated as

$$\begin{aligned} E_{\text{rad}} &= \int_{-\infty}^{\hat{\sigma}_{\text{int}}^-} \langle T_{--}^f(\hat{\sigma}^-) \rangle d\hat{\sigma}^- \\ &= M + \lambda C - \frac{\kappa\lambda}{4} \left[\ln\left(\frac{\kappa}{4}\right) - 1 \right] - \frac{\kappa\lambda\Delta}{4x_{\text{int}}^-}. \end{aligned} \quad (43)$$

The ADM mass of the dynamical solution (15) (relative to the reference solution with $C = C_0$) is $M_{\text{ADM}} = M + \lambda(C - C_0)[2]$. Thus we see that the unradiated mass δM remaining as $x^- \rightarrow x_{\text{int}}^-$ is

$$\begin{aligned} \delta M &= M_{\text{ADM}} - E_{\text{rad}} \\ &= \frac{\kappa\lambda}{4} \left[\ln\left(\frac{\kappa}{4}\right) - 1 \right] - \lambda C_0 + \frac{\kappa\lambda\Delta}{4x_{\text{int}}^-}. \end{aligned} \quad (44)$$

From the constraint equations (8) we find the thunderpop[2, 5]

$$\begin{aligned} [T_{--}^f(\hat{\sigma}^-)]_{\text{cl}} &= \frac{1}{2} \sum_{i=1}^N (\partial_- f_i)^2 \\ &= \frac{\kappa\lambda\Delta}{4x_{\text{int}}^-} \delta(\hat{\sigma}^- - \hat{\sigma}_{\text{int}}^-). \end{aligned} \quad (45)$$

The mass remaining after the shock wave (45) is $\frac{\kappa\lambda}{4} \left[\ln\left(\frac{\kappa}{4}\right) - 1 \right]$ when $C_0 = 0$, and would be mass M_w of the wormhole. Thus we try to match at intersection points (19) the solution (15) to one of the static wormhole solution,

$$e^{-2\phi} = e^{-2\beta} = -\lambda^2(x^+ + A)(x^- + B) - \frac{\kappa}{4} \ln[-\lambda^2(x^+ + A)(x^- + B)] + \frac{\kappa}{4} \left(\ln \frac{\kappa}{4} - 1 \right), \quad (46)$$

where $A = 0$, $B = \Delta$ and only the regions of $-\lambda^2 y^+ y^- = -\lambda^2(x^+ + A)(x^- + B) \geq 0$ is considered (see Figure 1). The solution (46) satisfies the constraints for wormhole that we mentioned below Eq.(36) since $\frac{\kappa}{4}(\ln \frac{\kappa}{4} - 1) > -1$ when $\frac{\kappa}{4} < 1$, and has no horizons and singularities in the wormhole spacetime, $-\lambda^2(x^+ + A)(x^- + B) \geq 1$.

One get the scalar curvature, $R = 8e^{-2\beta} \partial_{x^+} \partial_{x^-} \beta$, which is negative in the entire region of the wormhole spacetime except the asymptotically flat region. The no-positive property of the scalar curvature is required for an appropriate shape of the wormhole.

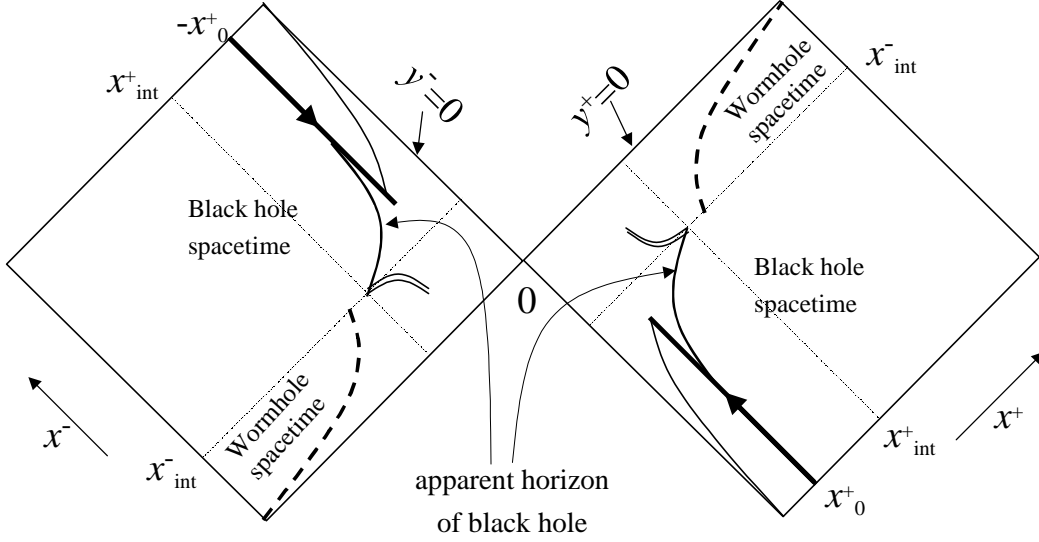


Figure 1: The (extended) Penrose diagram describing the evaporating black hole remaining the static wormhole at the end state. In this figure, $y^+ = x^+ + A$ and $y^- = x^- + B$. The gap between the horizon of the evaporating black hole and the throat of the wormhole would be caused by the thunderpop at the intersection point $(x_{\text{int}}^+, x_{\text{int}}^-)$. Two throat curves of the wormhole are identified in the opposite direction.

At the throat of the wormhole,

$$e^{-2\beta}|_{\text{at throat}} = e^{-2\Phi}|_{\text{at throat}} = 1 + \frac{\kappa}{4}(\ln \frac{\kappa}{4} - 1) \quad (47)$$

and the scalar curvature is also finite and negative,

$$R|_{\text{at throat}} = \lambda^2 \kappa \frac{1 - \frac{\kappa}{4} \ln \frac{\kappa}{4} - \frac{\kappa}{4}}{1 + \frac{\kappa}{4} \ln \frac{\kappa}{4} - \frac{\kappa}{4}} < 0. \quad (48)$$

In asymptotic region, one can get easily $\Phi = 0$ and $R = 0$.

In this work we introduced the extended light-cone coordinates and showed that there are boundary conditions in which an evaporating black hole remains a static wormhole as a candidate of end states of a evaporating black hole. These conditions preserve energy conservation and continuity of the metric. This final geometry from the black hole evaporation can avoid the information problem.

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